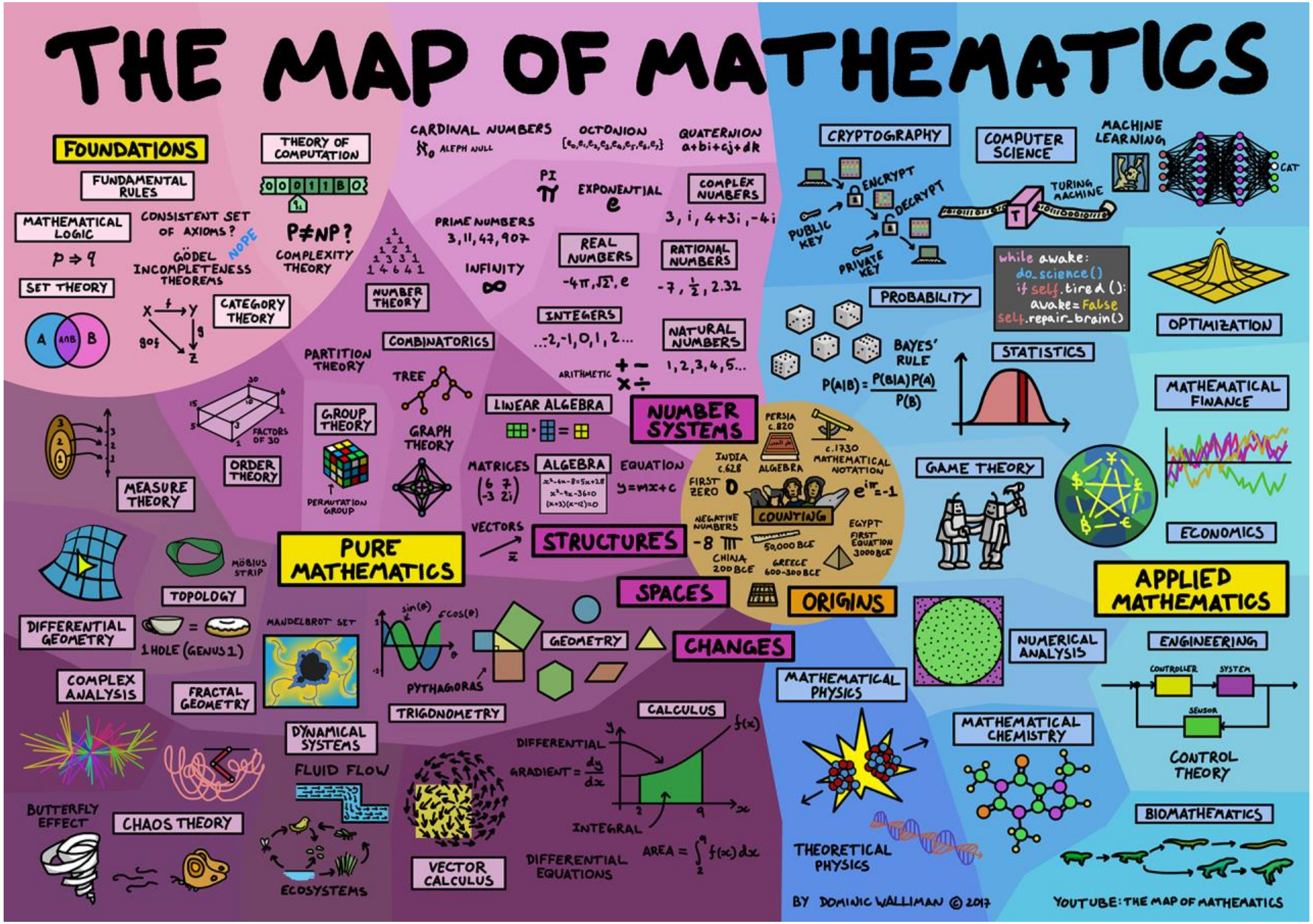
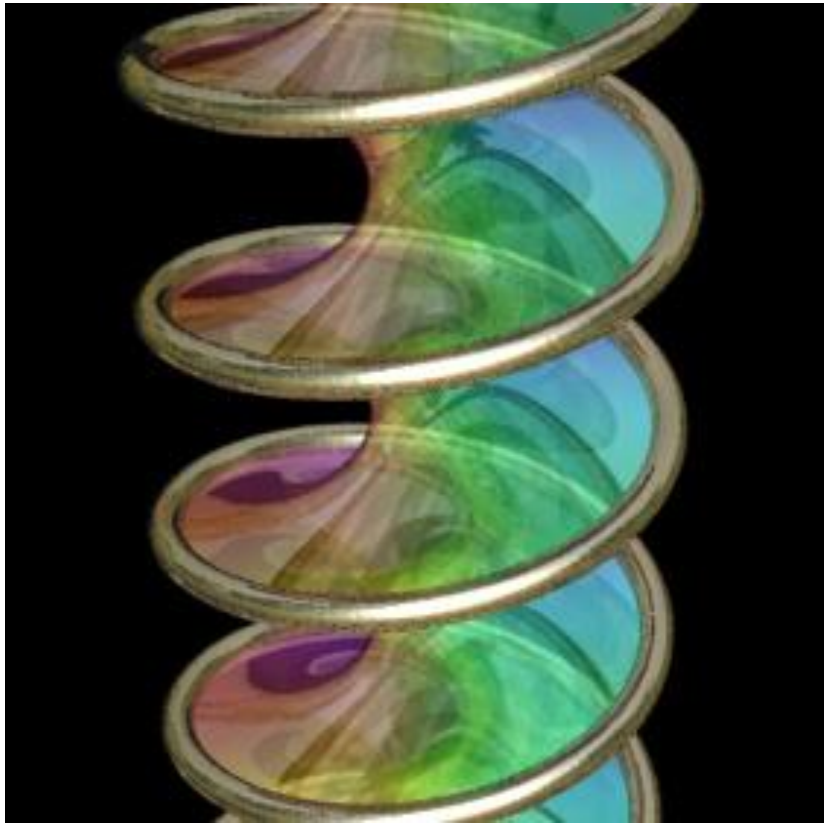
MILNI MEHURČKI, TRULLI, URE IN KATEDRALE

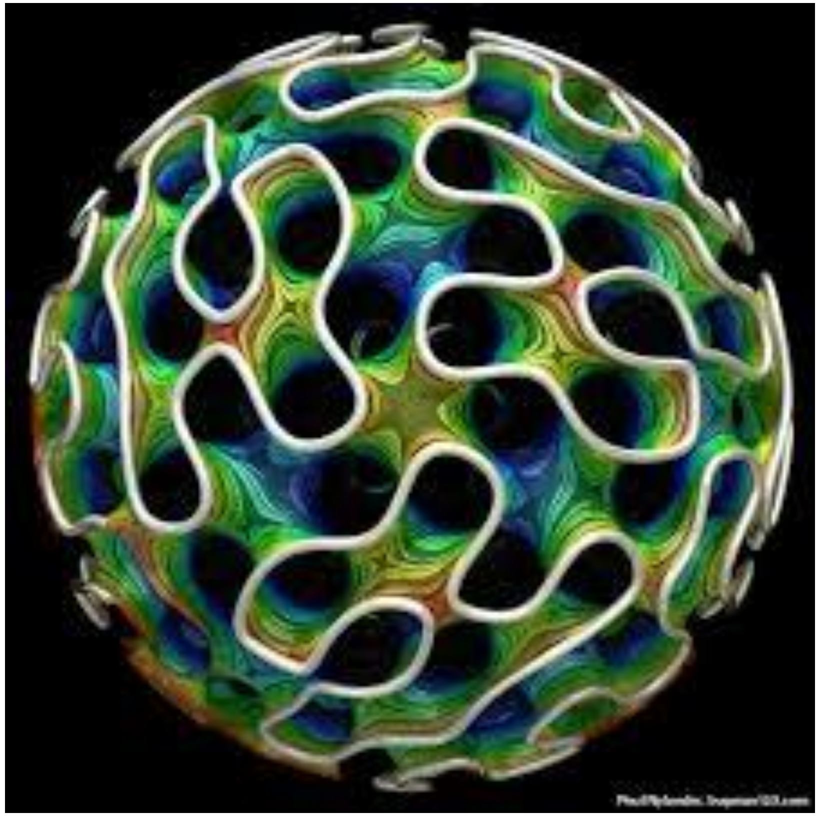
OPTIMIZACIJSKI PROBLEMI V MATEMATIKI




```
while awake:
    do_science()
    if self.tired():
        awake = False
self.repair_brain()
```

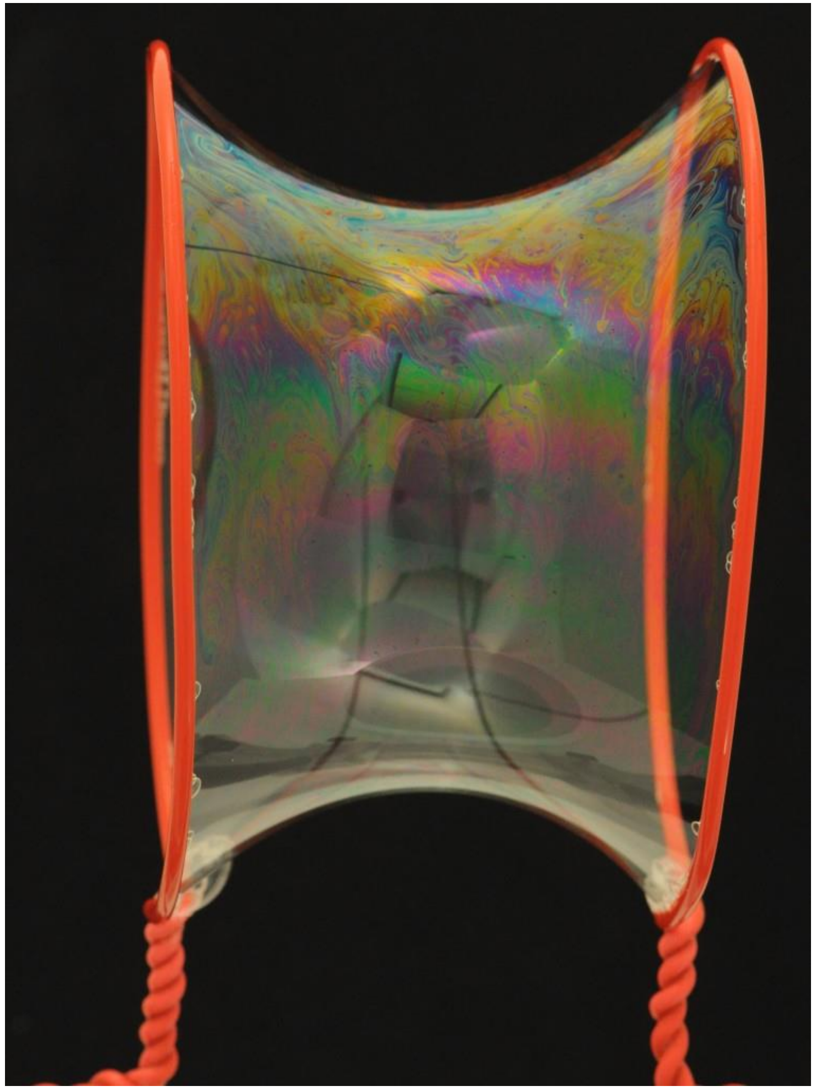
Milni mehurčki





Helikoida

Vir: bugman123.com/index.html

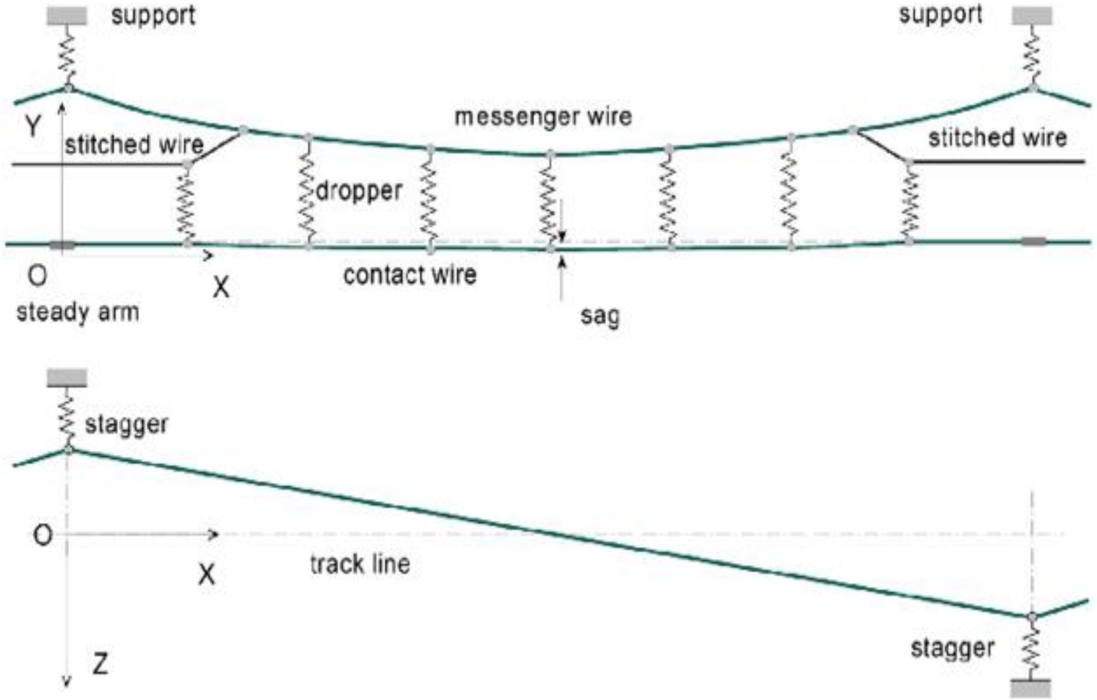


Katenoida

$$F(y) = 2\pi \int_{-a}^{a} y(x)\sqrt{1+y'^2(x)}dx$$

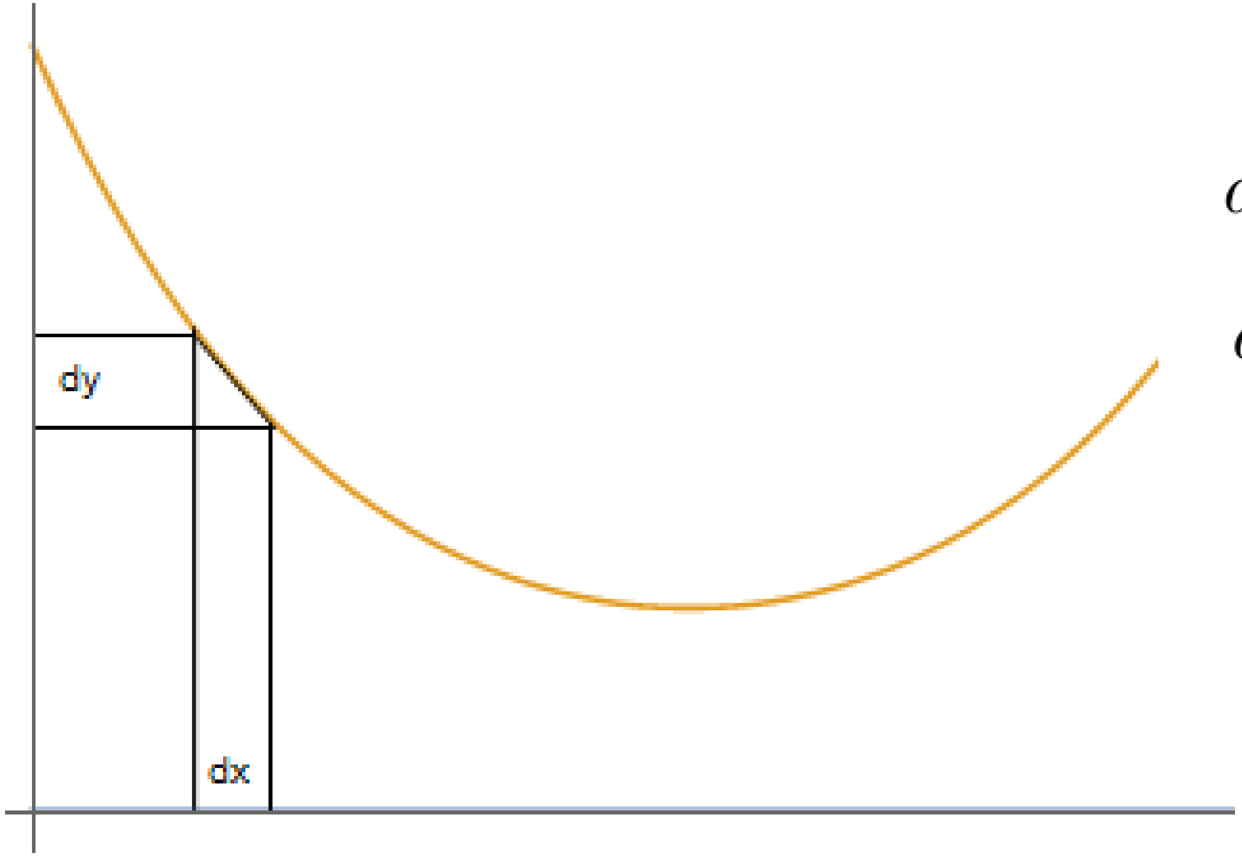
https://www.youtube.com/watch?v=_2DRRDndyD0

Verižnica



support
support
messenger wire
Y
stitched wire
stitched wire
dropper
O
X
contact wire
steady arm
sag
stagger
O
X
track line
Z
stagger

Minimizacija potenciala

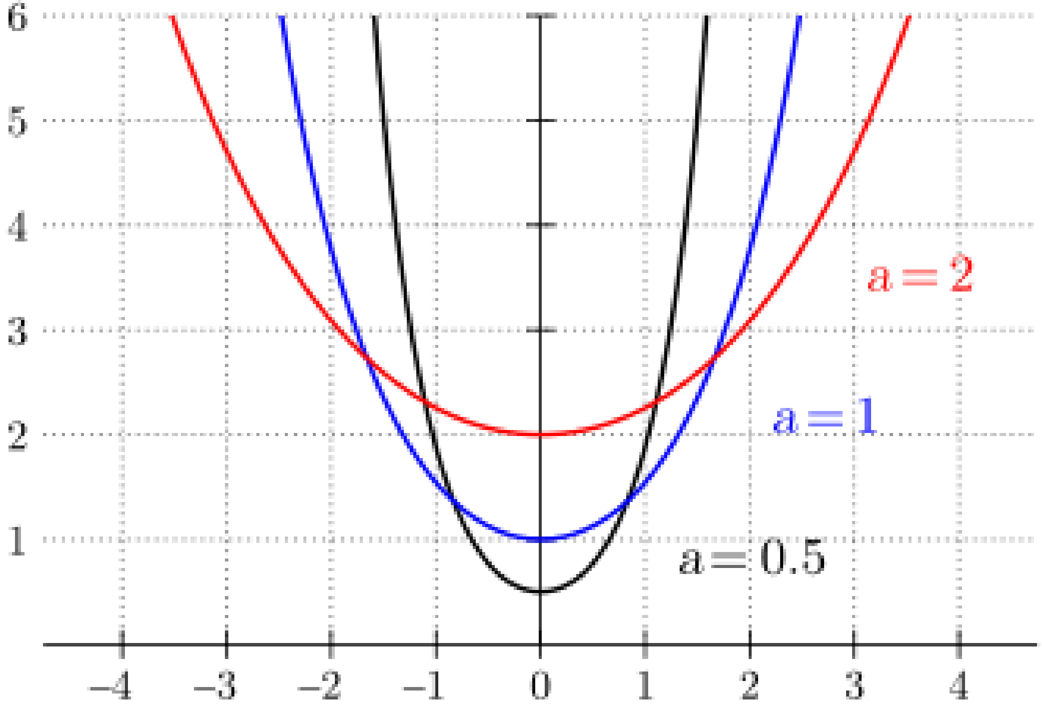


$$dP(x) = y(x)\sqrt{dx^2 + dy(x)^2}$$

$$dP(x) = y(x)\sqrt{1 + y(x)'^2}dx$$

$$F(y) = \int_0^c y\sqrt{1 + y'^2}dx$$

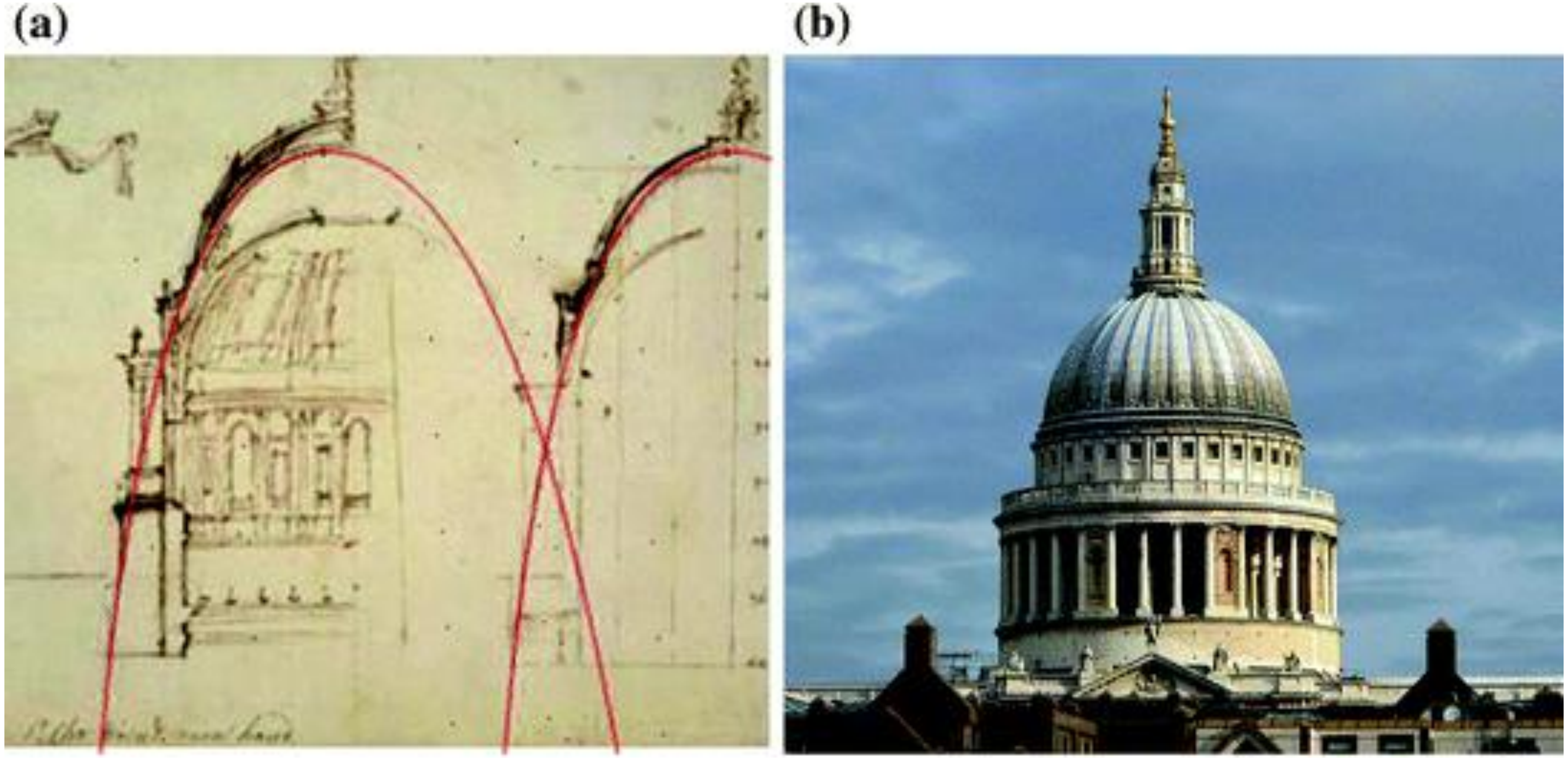
Matematična verižnica, y(x) = a ch(x/a)



Obok v St. Louisu

(RudyBalasko/iStock)

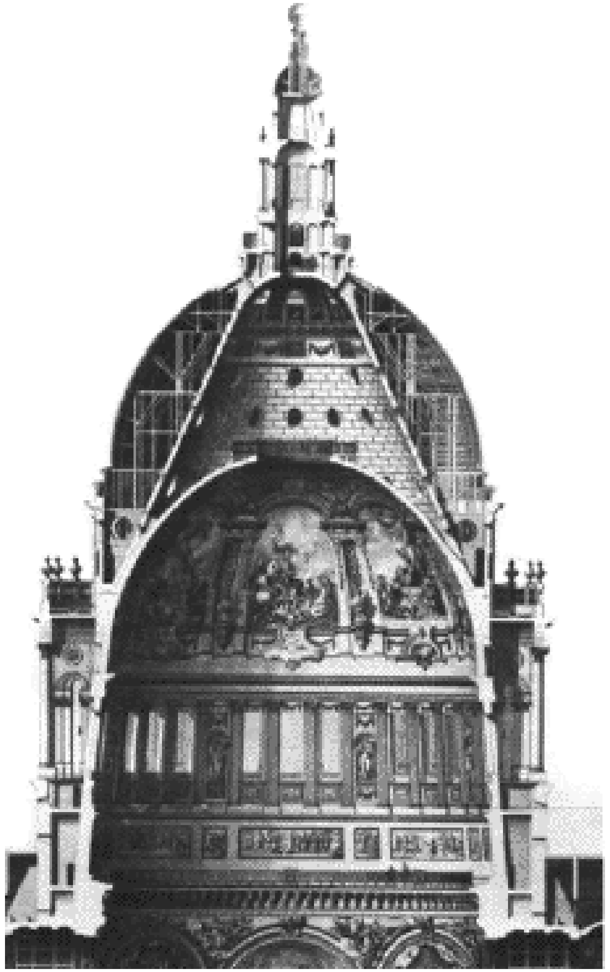
Katedrala svetega Pavla, London



Hooke, Wren, Brunelleschi

Vir: Wikipedia.org

- Hooke: stabilen lok je na glavo obrnjena verižnica, 1670
- model za lok: $\mathrm{y} = x^2$
- model za kupolo: $\mathrm{y} = x^3$
- Bernoulli, Huygens, Leibniz: rešitev 1690

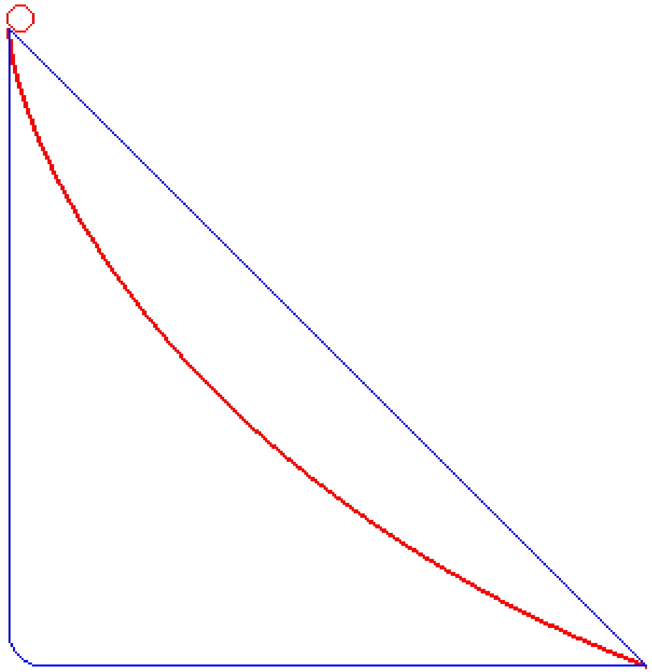


Vir: Wikiwand

Trulli

Brahistokrona

https://www.mathcurve.com/courbes2d.gb/brachistochrone/brachistochrone.shtml



Izpeljava funkcionala

$$\frac{1}{2}mv^2 - mgy = 0$$

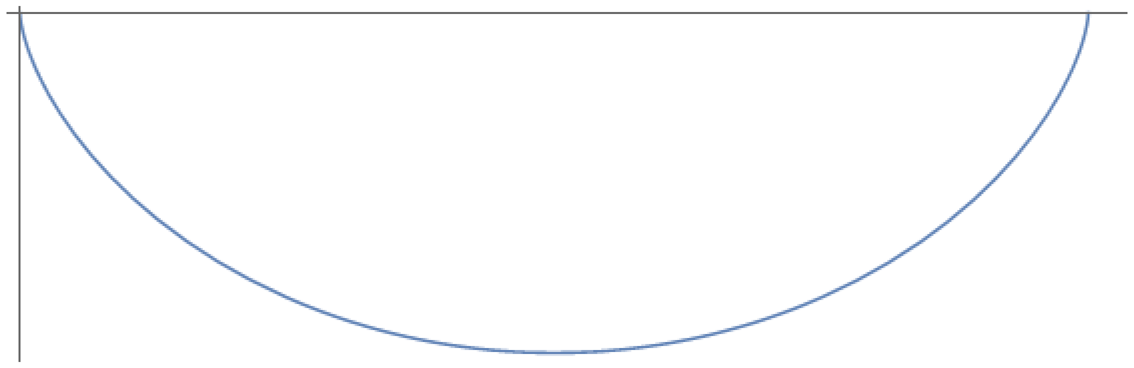
$$v = \frac{ds}{dt}, \qquad ds = \sqrt{1 + y'^2}dx, \qquad dt = \frac{ds}{v} = \frac{ds}{\sqrt{2gy}}$$

$$t = \int_0^b \frac{\mathrm{d}s}{v} = \int_0^b \frac{ds}{\sqrt{2gy}},$$

$$F(y) = \int_0^b \sqrt{\frac{1 + y'^2}{2gy}}dx$$

Brahistokrona je cikloida, dana s parametrizacijo

$X = R(p - \sin p)$, $Y = -R(1 - \cos p)$



Tavtokrona

Če se kroglica giblje po tavtokroni, doseže najnižjo točko v nihajnem času, ki ni odvisen od amplitude njenega nihanja. Huygens je dokazal, da je to cikloida. Na ta način lahko tavtokrono uporabimo za merjenje časa.

Vir: Wikipedia

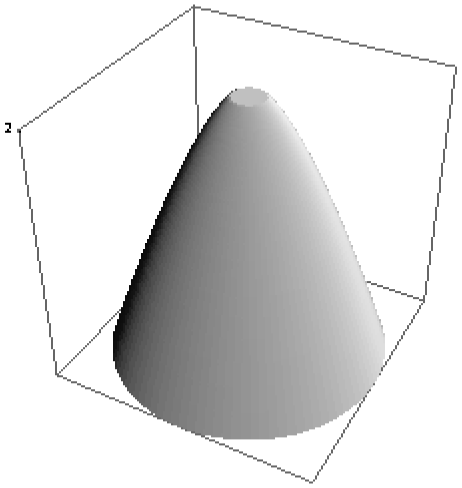
Ploskve z minimalnim uporom

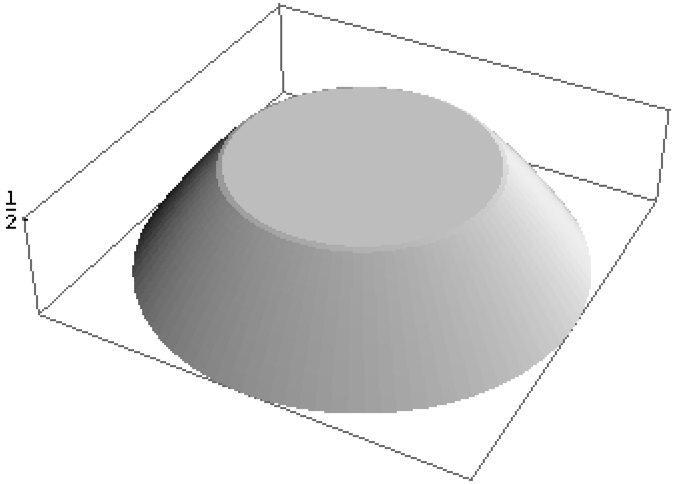
Ploskev z minimalnim uporom višine 2 H in dolžine l je minimum funkcionala

$$F(y) = \frac{H^2}{2} + \int_0^l \frac{yy'^3}{1+y'^2} dx$$

Vir: Wikipedia, Newton's minimal resistance problem

Primeri ploskev z minimalnim uporom





Vir: TU/e -> CASA -> Mark Peletier

Didin problem (Vergil, Eneida, 1. knjiga)

His commota fugam Dido sociosque parabat:
conveniunt, quibus aut odium crudele tyranni
aut metus acer erat; navis, quae forte paratae,
corripiunt, onerantque auro: portantur avari
Pygmalionis opes pelago; dux femina facti.
Devenere locos, ubi nunc ingentia cernis
moenia surgentemque novae Karthaginis arcem,
mercatique solum, facti de nomine Byrsam,
taurino quantum possent circumdare tergo.

Dido je s trakovi iz bikove kože obkrožila celoten hrib, ki ima danes ime Byrsa (bikova koža po grško)

Vir: Wikipedia.org

Izoperimetrični ali Didin problem

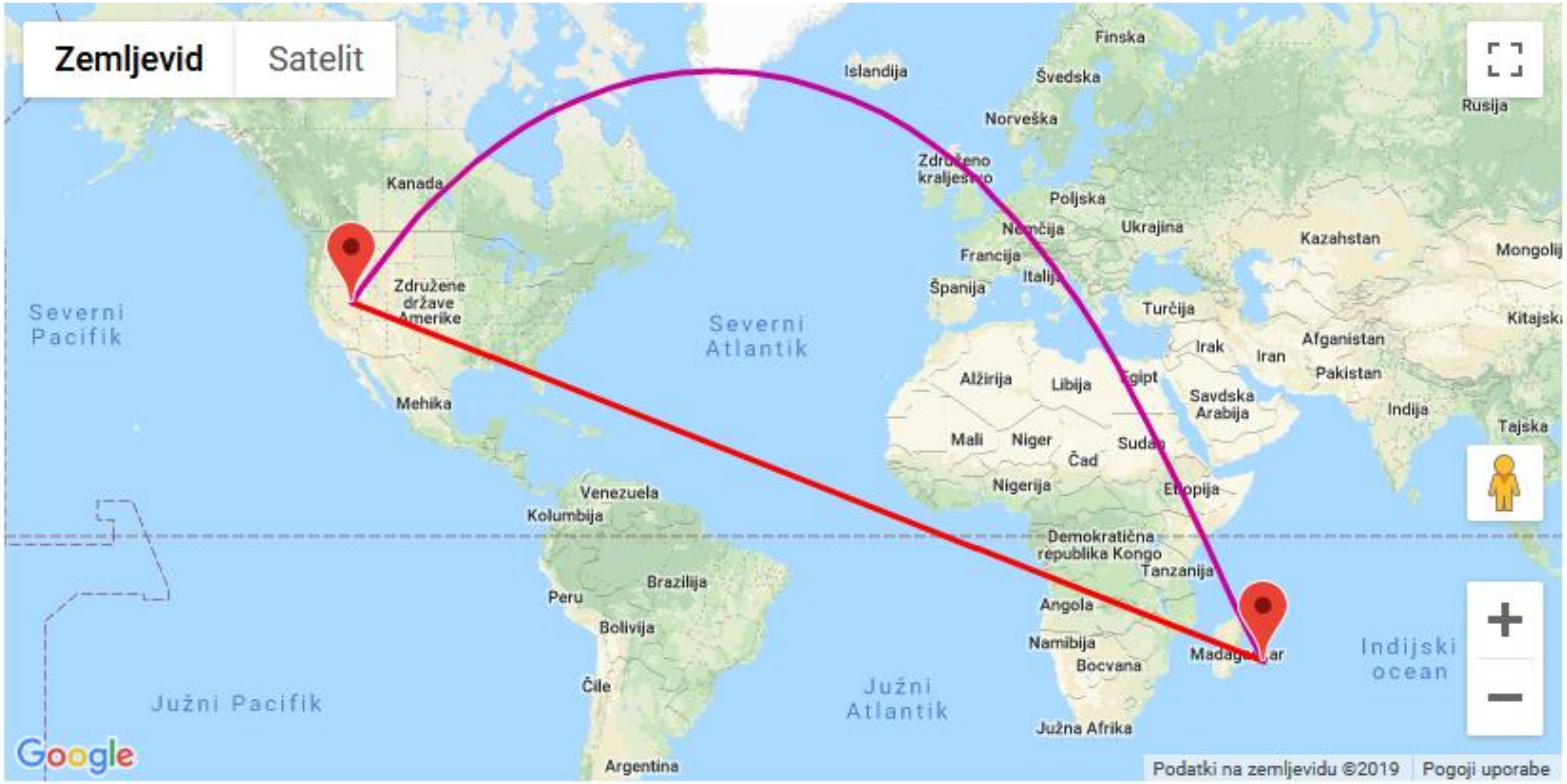
Minimiziramo F pri pogoju G = c

$$F(y) = \int_a^b y(x)\, dx$$

$$G(y) = \int_a^b \sqrt{1 + y'(x)^2}\, dx = c$$

Rešitve so krožnice.

Najkrajše poti (geodetke) na Zemlji



Vir: © Academo.org 2016, google maps

Izpeljava formul

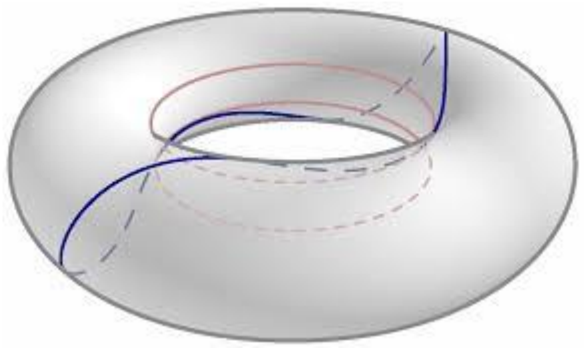
$$\gamma(x) = (x, y(x), \sqrt{R^2 - (x^2 + y^2)})$$

$$\dot{\gamma}(x) = (1, y', (-x - yy')/\sqrt{R^2 - (x^2 + y^2)})$$

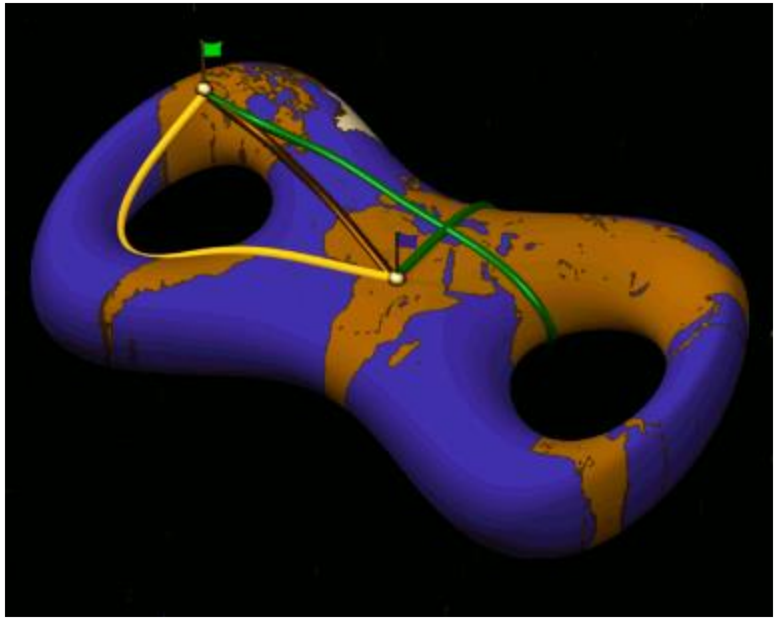
$$ds = (1 + y'^2 + (x + yy')^2/(R^2 - (x^2 + y^2(x)))$$

$$F(y) = \int (1 + y'^2 + (x + yy')^2/(R^2 - (x^2 + y^2))\, dx$$

Geodetke na drugih ploskvah



Irons, Mark L. "The curvature and geodesics of the torus." 2008.



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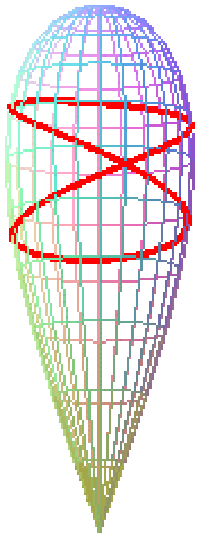

www.mathcurve.com/ surfaces.gb/tannery/tannery.shtml

Hvala za pozornost!