# P versus NP
## A million-dollar question (and more)

Clément Dallard

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Izleti v matematično vesolje

Univerza na Primorskem

November 24, 2021




# AHHH ... SWEET 60S ...


Source: `computerhistory.org`

# AHHH ... SWEET 60S ...


Source: `flickr.com`, ArnoldReinhold

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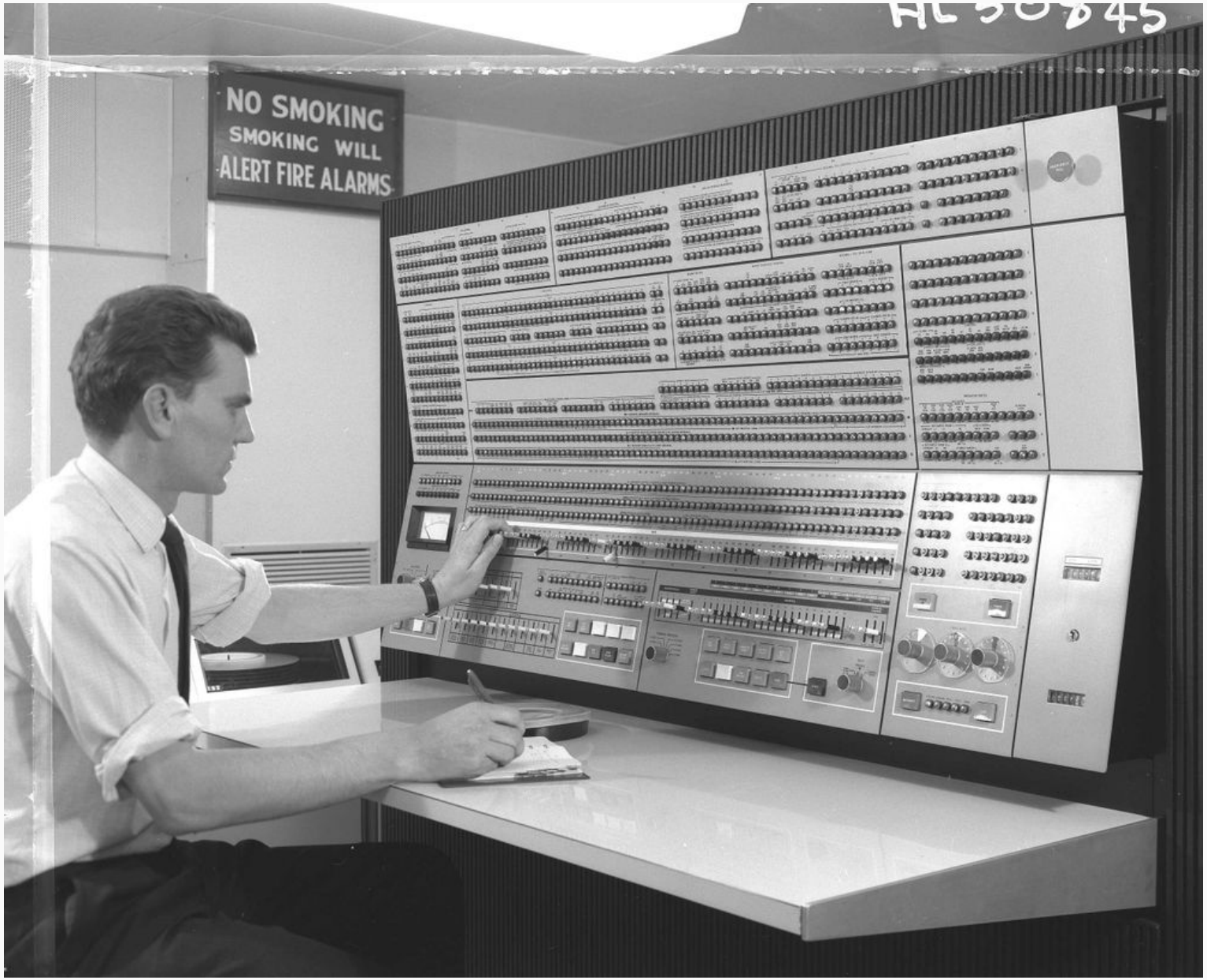


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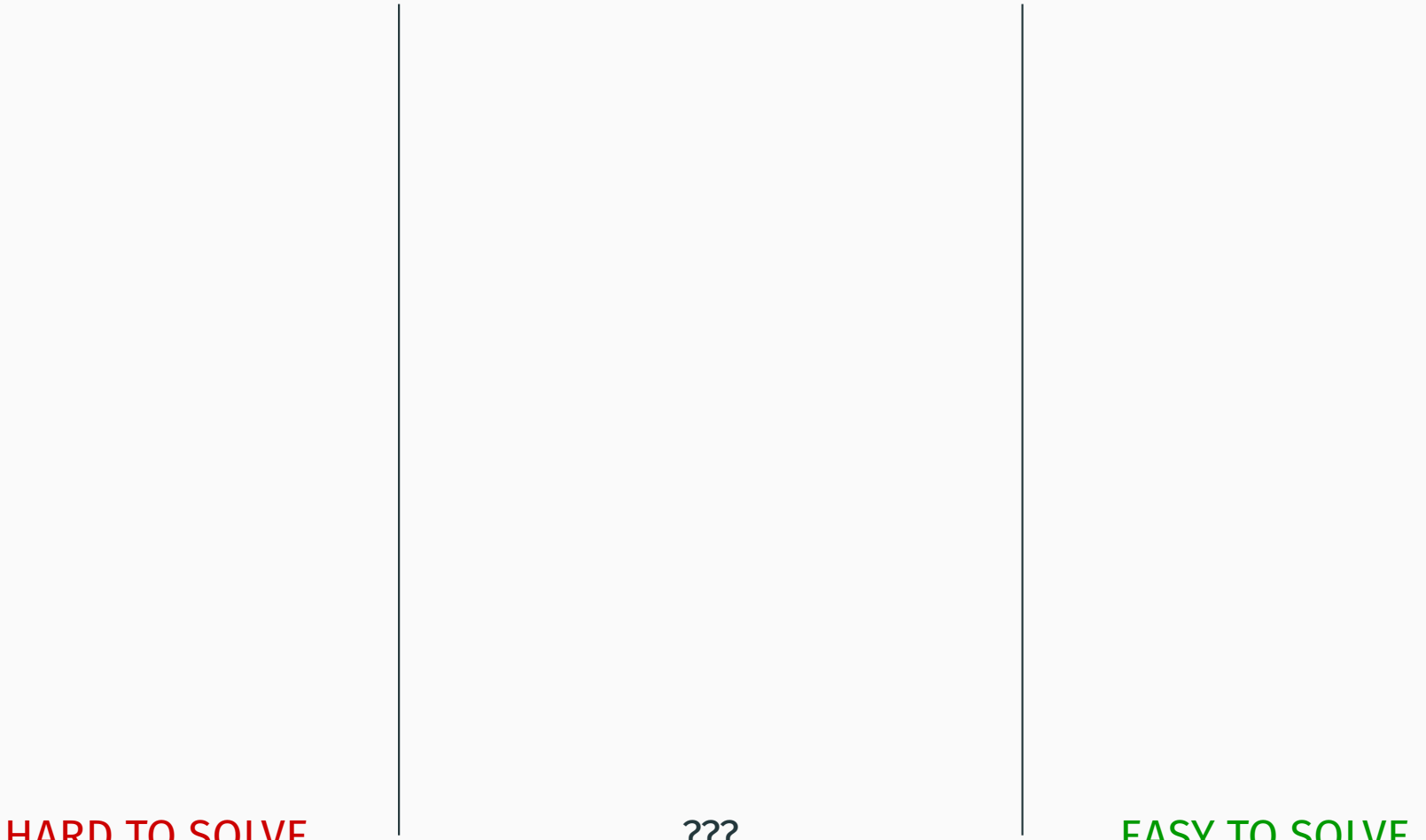
# HARD OR EASY, THAT IS THE QUESTION



# Hard or easy, that is the question

Multiplication

Shortest Path

Primality Testing

Sorting

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| hard to check a solution | easy to check a solution | easy to check a solution |

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The **P** versus **NP** question asks whether **finding** a solution is as easy as **checking** a solution.

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An efficient algorithm for an **NP**-complete problem would provide an efficient algorithm for **every** problem in **NP**, therefore showing that $\mathbf{P} = \mathbf{NP}$.

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Let $\phi = (x_1 \text{ OR } x_2) \text{ AND } (\bar{x_1} \text{ OR } x_2 \text{ OR } x_3) \text{ AND } (x_1 \text{ OR } \bar{x_3}) \text{ AND } (\bar{x_2} \text{ OR } \bar{x_3})$.
Then $\phi$ can be satisfied by setting $x_1 = \text{true}$, $x_2 = \text{true}$, and $x_3 = \text{false}$.

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Nowadays, thousands of problems are known to be **NP**-complete.

We only know **exponential** algorithms for **NP**-complete problems: if we knew a **polynomial** (that is, **efficient**) algorithm, then we would know that $\mathbf{P} = \mathbf{NP}$.

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- Sudoku: NP-complete (Yato, 2003)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | | | 7 | | | | |
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
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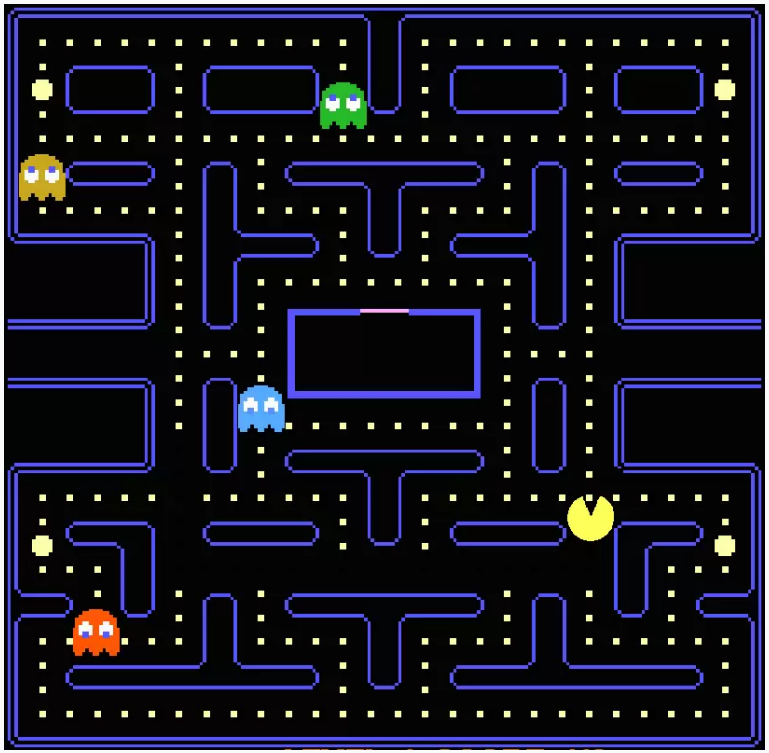

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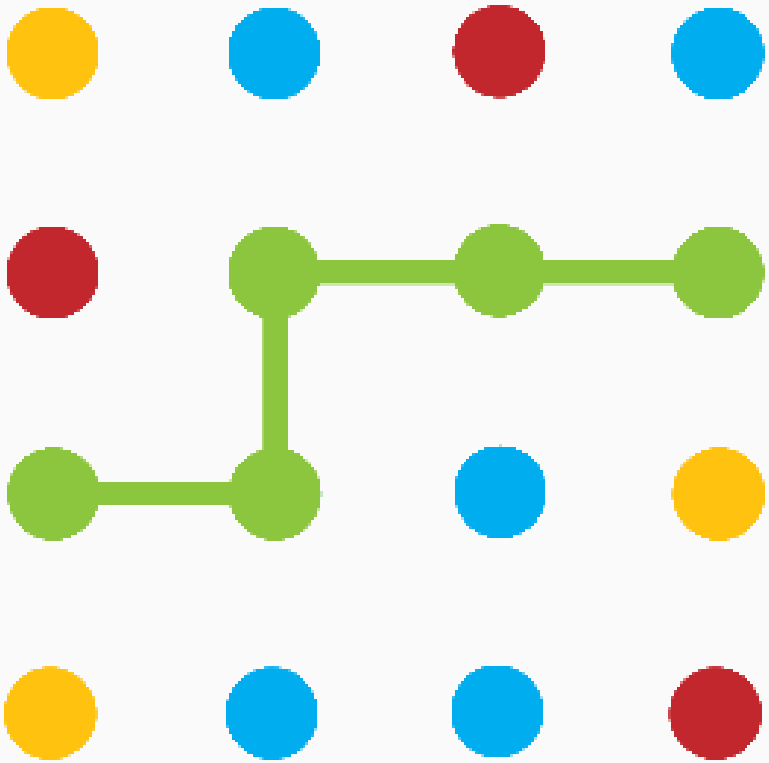

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- Angry Birds: NP-complete
  (Stephenson, Renz, and Ge, 2017)

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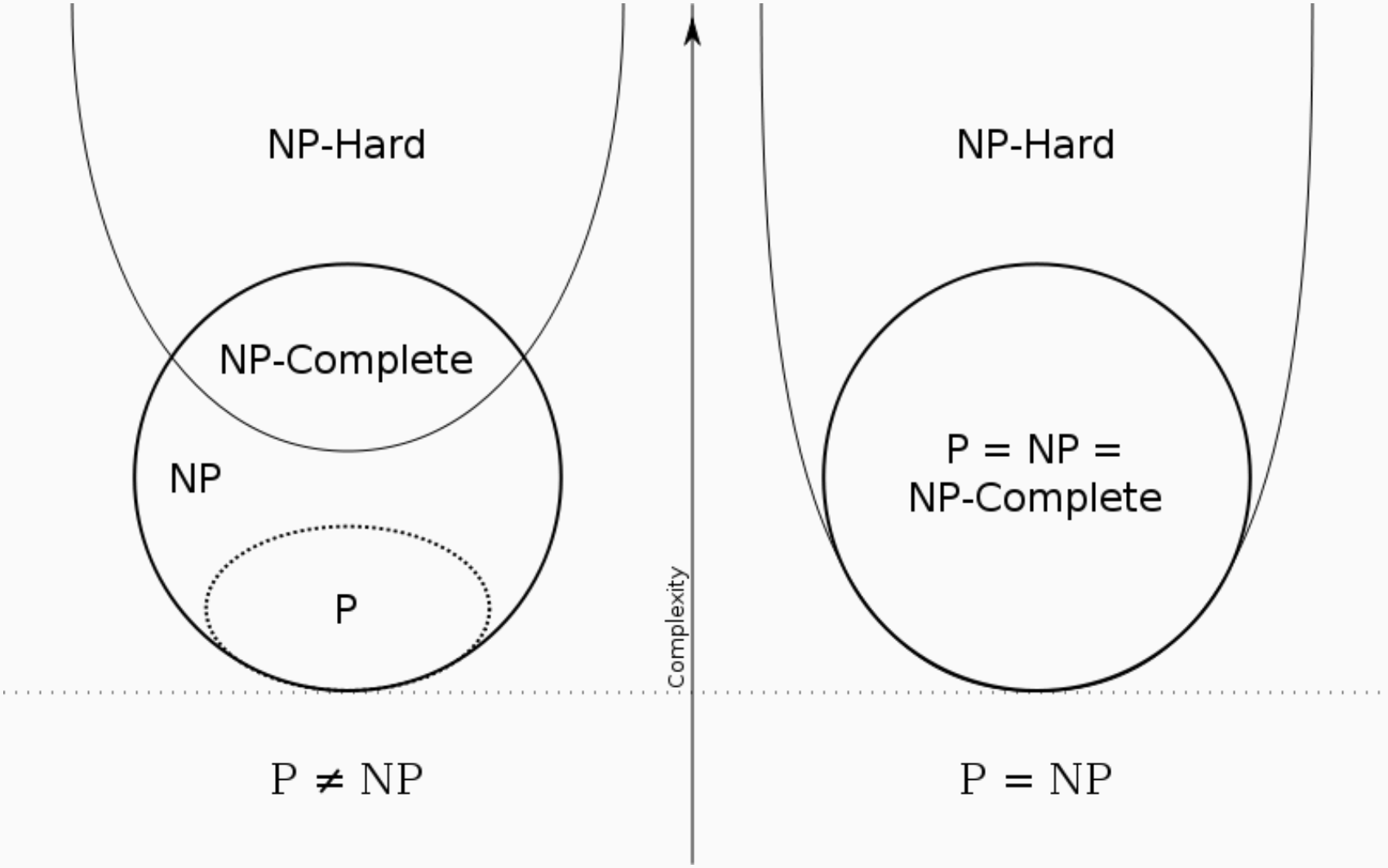

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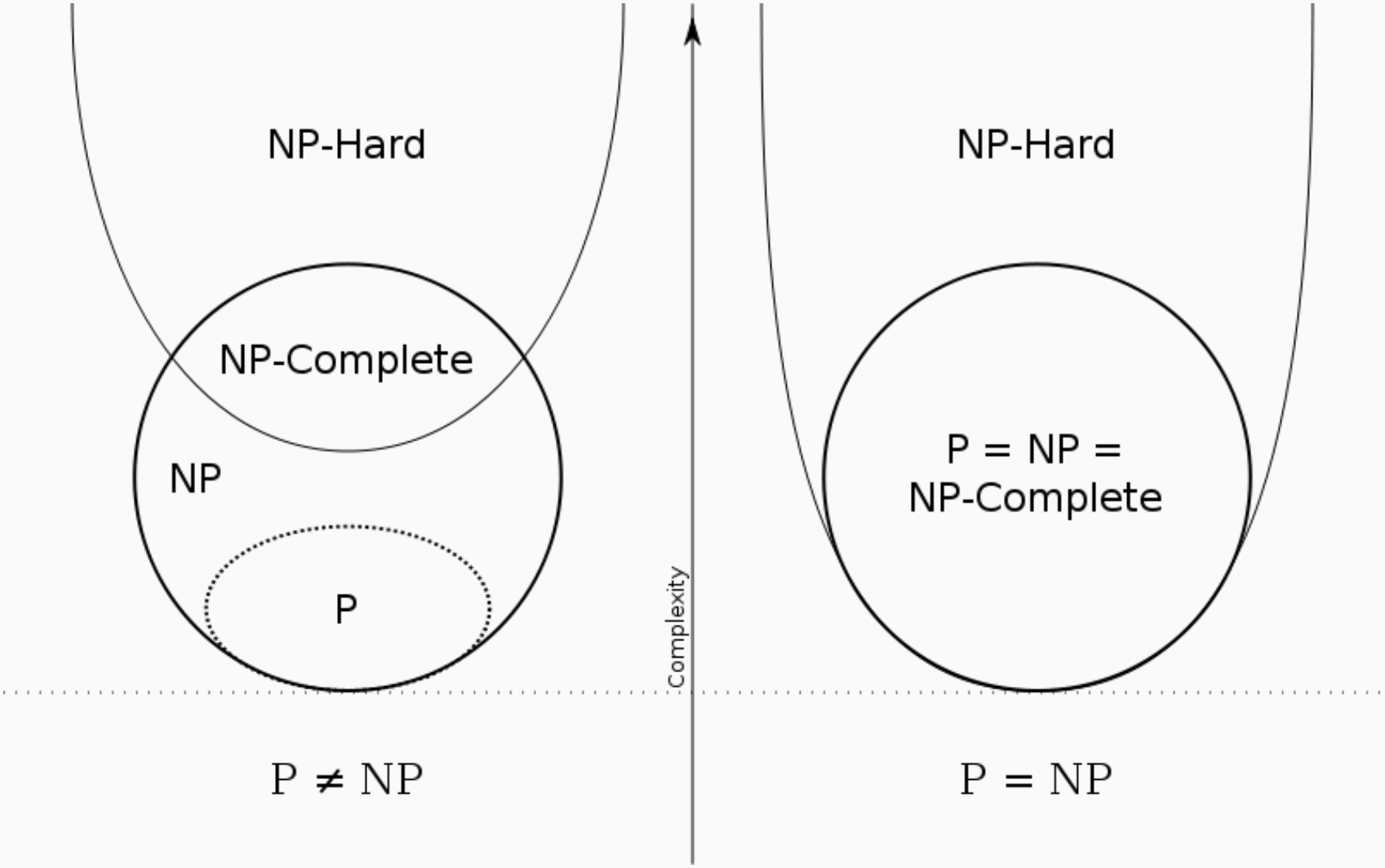


If **P** $\neq$ **NP**, then we know that there are problems in **NP** that are neither **NP**-complete nor in **P** (Ladner, 1975).

# You said one million dollars?

**Millenium prize problems** (Clay institute):

- Yang–Mills and Mass Gap
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If you prove that $\mathsf{P} = \mathsf{NP}$ by providing an efficient algorithm for an NP-complete problem, don't say anything to anyone (except me).

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- nothing else really changes, but we should be ready to update our encryption algorithms!

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We would feel better when we don't finish a Sudoku grid!

The NUMBER SCRABBLE game:

- two players take turn to select numbers from 1 to 9 without repeating any numbers already selected;

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Player 1:

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|---|---|---|---|---|---|---|---|---|---|
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If displayed as a magic square:

| | | |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |

# SUPPORTING P = NP

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Besides, some problems have been proved to belong to **P** ony after several years of research ...

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> *Given how many confusions and weird misinterpretations it's spawned, I, Scott Aaronson, hereby formally disown my statement about how "everyone who could appreciate a symphony would be Mozart" if $P = NP$.*
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# Can we even prove it?

Maybe the **P** versus **NP** question simply not provable in any of the current logical theories.

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- **Heuristics**: don't compute an optimal solution, just try to not be too bad.
- **Specific inputs**: develop efficient and optimal algorithms for specific kind of inputs.
- **Improve the base of the exponent**: $1.234^n$ grows way slower than $2^n$.

# QUANTUM COMPUTING TO THE RESCUE?

**BQP** is the class of problems that **can be solved efficiently** with an error probability of at most 1/3 (for all inputs) **on a quantum computer**.

(**BQP** stands for "**B**ounded-error **Q**uantum **P**olynomial time.")

# QUANTUM COMPUTING TO THE RESCUE?

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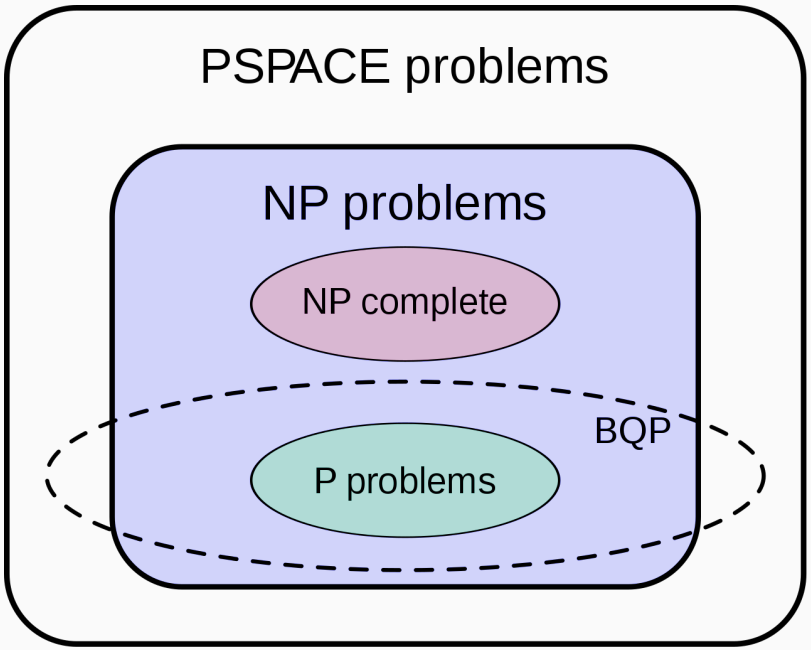
- It is believed that $\mathbf{NP} \not\subseteq \mathbf{BQP}$.




Suspected relationship of **BQP** with **NP**

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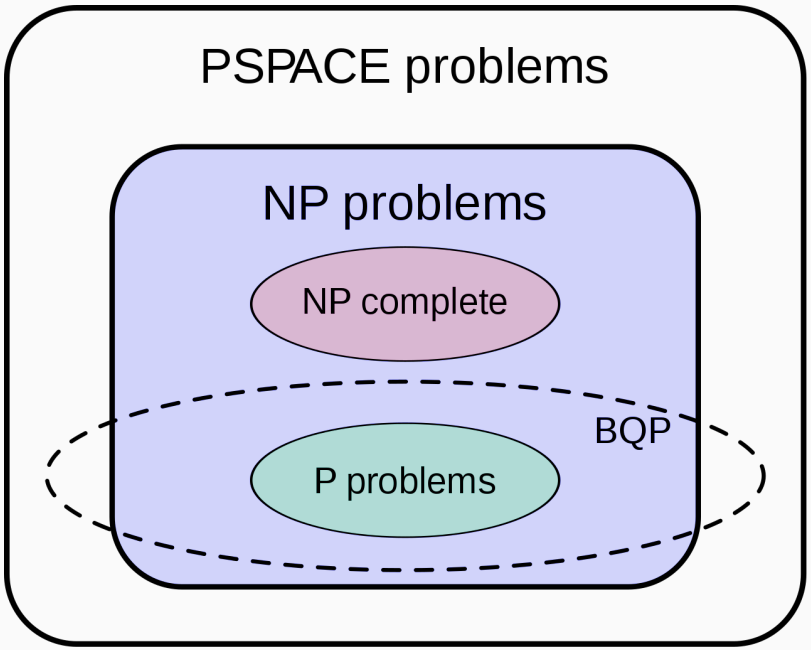
- We know that $\mathbf{P} \subseteq \mathbf{BQP}$.
- It is believed that $\mathbf{NP} \not\subseteq \mathbf{BQP}$.
- We can solve some problems efficiently on quantum computers, including PRIME FACTORIZATION, but no **NP**-complete problem.

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# Thank you!

Questions?