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Paper notes from 23 Mar 2022 Lecture

$$6x^1 = x\cdot1\cdot1\cdot1\cdot1\cdot1 + 1\cdot x\cdot1\cdot1\cdot1\cdot1 + 1\cdot1\cdot x\cdot1\cdot1\cdot1 + \dots$$

Revisit 15:

$r=13$ divides $15!$

but not $1!, 2!, 3!, 4!, \dots, 12!$

So 13 divides $\binom{15}{3}, \dots \binom{15}{12}$

We need only find p dividing

$\binom{15}{1}=15$ and $\binom{15}{2}=105$

3 or 5 "work"

$$(1+x)^2 \equiv_2 1+x^2$$
$$\parallel$$
$$1+2x+x^2$$

$$(1+x)^{p^2} = (1+x)^{p\cdot p} \equiv_p (1+x^p)^p \equiv_p 1+x^{p^2}$$

$$(1+x)^{15} \equiv_{13} (1+x)^{13}\cdot(1+x)^2$$
$$\equiv_{13} (1+x^{13})\cdot(1+2x+x^2)$$
$$\equiv_{13} 1+2x+x^2+x^{13}+2x^{14}+x^{15}$$

While

$$(1+x)^{15} \equiv_5 (1+x)^{5\cdot3} \equiv_5 (1+x^5)^3$$
$$\equiv_5 1+3x^5+3x^{10}+x^{15}$$

Ex: $n = 210$. Largest prec. prime is 199
But 103 is prime.

$$(1+x)^{210} \equiv_{103} (1+x^{103})^2 \cdot (1+x)^4$$

$$\equiv_{103} (1+2x^{103}+x^{206}) \cdot (1+4x+6x^2+4x^3+x^4)$$

$$\equiv_{103} (1+4x+\ldots+x^4+x^{103}+4x^{104}+6x^{105}+\ldots$$

$$(1+x)^{210} \equiv_7 (1+x^{7^2})^4 (1+x^7)^2$$

nonzero coef at x^{98}, $\boxed{x^{105}}$

$$(1+x)^{210} \equiv_5 (1+\cancel{8}x^{5^3})(1+x^{5^2})^3 \cdot (1+x^5)^2$$

zero coefs around x^{105}

deals w/ coefs left by 103.